**IE 517 HW2**

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**1.Problem Introduction**

P-median problem is a specific type of a discrete location problem. The aim of the problem is minimizing the demand weighted distance between the customers and facilities, which is an approximation of the total delivery cost. In this setting, there are no capacity constraints. Two metaheuristics, namely Simulated Annealing and VNS(Variable Neighborhood Search) are implemented to obtain the solutions. For the numerical analysis, 3 different customer locations and 3 different p values are tested with 10 different initial solutions. In this setting, customer locations are defined also as potential facility locations. In the Section-2 and Section-3, Simulated Annealing and VNS heuristics will be explained, respectively. In the Section-4, results of the numerical analysis will be discussed with given summary tables for each instance.

**2. Simulated Annealing**

In the implementation of the simulated annealing algorithm, the general structure of the algorithm by Liu et. al. [1] is taken as reference and modified to our current problem setting. 3 different customer locations with 3 different p values (total of 9 instances each with 10 different initial solutions) are tested and the results including final solution, objective value, number of iterations, total CPU time are shown in summary tables in Section-4. Each table name refers to the total number of customers and p-values, respectively. (e.g., 51-4 refers to total of 51 customer locations and 4 facilities.)

**2.1. Solution Representation**

Discrete representation is preferred in this problem setting. A list of length p is used and each element in the list refers to the location of the facility opened. For instance, [4,17,24,34] means that 4 facility is opened in the customer locations 4,17,24 and 34. For the neighborhood search, 1-swap move operator is used. This operator removes one of the existing facilities and opens a new facility in another location randomly. An example is given below:

[4,17,24,34] -> [4,17,28,34]

Here, move operator removes facility at 24 and, instead, add a new facility at 28 at random. In implementation of the algorithm, there are four parameters that need to be defined beforehand.

In the parameter selection, design of experiment methodology is used to obtain the most promising parameter combinations given a pool of candidate parameter values. The candidate parameter values are taken from [1], and the best set of parameters are used in the implementation of the algorithm as a result. In the code section, the parameters which give best result after the experimentation process are used, and parameter selection process is omitted. The name of the parameters are r,sf,ip and fp. r and ip is used in the cooling process and decides on the degree and the conditions of the cooling process. Sf is used to determine the run length L at each temperature. fp is important in the stopping condition of the algorithm. A counter, terct, is incremented by one each time the run length at a temperature has been reached for which the percentage of accepted moves is fp or less and is reset to 0 each time the percentage of accepted moves of a temperature is larger than fp. Initial temperature is set to 100000. Time limit is used 10 minutes as another stopping condition, in the case of which the algorithm does not end in a considerable amount of time with the current setting. The numerical results of the algorithm will be discussed in Section-4.

**3. Variable Neighborhood Search**

In the implementation of the VNS, *1-Swap, 2-Swap,3-Swap* moves are used in the shaking phase. In the local search phase, *1-Swap* move operator is used. The function of 1-Swap operator is the same as of the operator used in Simulated Annealing. Similarly, 2-Swap operator removes 2 existing facilities and adds 2 new facilities at random. 3-Swap operator functions the same, except this time with 3 facilities. Shaking phase contributes to *diversification* and, conversely, local search contributes to *intensification* of the solution search procedure.

The high-level pseudocode of the VNS algorithm implemented is as follows:

1. Initialization: Neighborhood structure Nk{k=1,2,3} is defined. Initial solution is obtained at random. Stopping condition is set as max CPU-time. (3-5-5 minutes respectively for each problem setting.)
2. **Repeat** the following **until** *stopping condition* is met:
3. Set k<-1, **Until** k=kmax **repeat**:
4. *Shaking*: Generate x’ at random from the k’th neighborhood of x.
5. *Local Search:* Apply local search x’ to obtain local optimum x’’.
6. *Move or not:* If this local optimum is better than the incumbent move there (x<- x’’) and continue the search with k<-1; otherwise set k<-k+1

**3.1. Solution Representation**

Discrete representation is preferred in this problem setting, similar to the one used in the Simulated Annealing algorithm. k-Swap move operator functions as follows:

*Remove* k existing facility randomly.

*Add* k new facility randomly.

For instance, an example with 2-Swap operator with p=6 is given below:

[6,12,21,45,18,67]-> [6,12,58,45,16,67]

Here, 2-Swap operator removes two facilities at locations 21 and 18 and adds two facilities at locations 58 and 16, randomly. There are two parameters defined in this algorithm: “kmax”, which refers to maximum number of neighborhood structure used and “stopping time”, which indicates the stopping condition of the algorithm. kmax is given as 3 in the problem description, and stopping time is taken as 3-5-7 minutes, for each of the problem setting. The numerical results of the algorithm will be discussed in Section-4.

**4. Numerical Results**

This section describes the computational experiments carried out on a machine with an M1 3.2 GHz processor and 8 GB of RAM. Both of the algorithms were coded in Python. 10 replications were performed for each instance and their results are summarized in “Summary Tables” given in the project description. The comment on the results is done in Section 4.1 and the summary tables are shown in Section 4.2. It is preferred to store initial solutions obtained from Simulated Annealing separately, later to be used in VNS algorithm. This way, same initial solutions are guaranteed to be used in both of the algorithms (Instead of random seeding). The global optimum values of the problem sets have been given beforehand:

|  |  |  |  |
| --- | --- | --- | --- |
|  | p=4 | p=6 | p=8 |
| eil51 | 7555,76 | 5836,88 | 4704,48 |
| eil76 | 23072,88 | 17705,87 | 15183,50 |
| eil101 | 54147,08 | 43341,09 | 36972,23 |

**4.1 Comment on the Results**

In all of the instances simulated annealing algorithm has found the global optimal solutions after 10 runs in relatively short time. However, as the total number of customer increases, the number of runs which can find global optimum tend to decrease, as expected. Similarly, total CPU time and average number of iterations increase proportionally with the total number of customers and facilities to be opened.

VNS algorithm also performed good, as it only failed to find the global optimum in the instance of 101-8,still, the gap between the global optimum and best solution is calculated as %0.03, which is quite low. The variance of the results between the runs is greater in VNS than of the Simulated Annealing. Also, the VNS algorithm takes more CPU time than Simulated Algorithm. It can be concluded that simulated annealing was, overall, more successful than VNS algorithm in this problem setting.

**4.2. Appendix: Summary Tables**

***4.2.1 Simulated Annealing Results***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Instance: SA:eil51-4 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (49,8,4,29) | 7555,76 | 3854 | 54,65592 |
| 2 | (49,8,4,29) | 7555,76 | 4136 | 57,66712 |
| 3 | (9,17,20,48) | 7703,67 | 4136 | 57,47972 |
| 4 | (8,49,20,4) | 7559,79 | 3290 | 45,64871 |
| 5 | (49,8,4,29) | 7555,76 | 5640 | 78,29257 |
| 6 | (49,8,4,29) | 7555,76 | 5640 | 573,5761 |
| 7 | (49,8,4,29) | 7555,76 | 4324 | 287,9626 |
| 8 | (49,8,4,29) | 7555,76 | 5264 | 73,75829 |
| 9 | (49,8,4,29) | 7555,76 | 4888 | 68,01948 |
| 10 | (49,8,4,29) | 7555,76 | 6204 | 86,26001 |
| Best Run | | |  |  |
| 1,2,5,6,7,8,9,10 | (49,8,4,29) | 7555,76 | Avg. No. Of iterations=4738 | Total CPU Time =1383,321 |

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| Instance: SA:eil51-6 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (8,9,20,37,19,14) | 5836,88 | 6210 | 86,89594 |
| 2 | (8,9,20,37,19,14) | 5836,88 | 4995 | 70,64136 |
| 3 | (14,8,49,19,29,17) | 5890,22 | 4185 | 58,34295 |
| 4 | (14,8,49,19,29,17) | 5890,22 | 4995 | 69,69629 |
| 5 | (8,9,20,37,19,14) | 5836,88 | 4995 | 69,6126 |
| 6 | (8,9,20,37,19,14) | 5836,88 | 5130 | 312,777 |
| 7 | (8,9,20,37,19,14) | 5836,88 | 5805 | 547,5639 |
| 8 | (8,9,20,37,19,14) | 5836,88 | 5535 | 77,41118 |
| 9 | (8,9,20,37,19,14) | 5836,88 | 5400 | 76,09226 |
| 10 | (14,8,49,19,29,17) | 5890,22 | 6075 | 84,67371 |
| Best Run | | |  |  |
| 1,2,5,6,7,8,9 | (8,9,20,37,19,14) | 5836,88 | Avg. No. Of iterations=  5333 | Total CPU Time =  1453,70719 |

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| Instance: SA:eil51-8 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (19,1,9,47,26,15,20,14) | 4704,48 | 5160 | 72,98416 |
| 2 | (14,50,10,26,35,1,19,17) | 4770,61 | 5332 | 74,97335 |
| 3 | (19,1,9,47,26,15,20,14) | 4704,48 | 6708 | 93,73953 |
| 4 | (47,1,26,14,9,20,33,42) | 4734,23 | 6364 | 89,90694 |
| 5 | (26,9,33,14,20,19,17,1) | 4732,68 | 7396 | 457,6102 |
| 6 | (19,1,9,47,26,15,20,14) | 4704,48 | 7224 | 593,4557 |
| 7 | (14,50,10,26,35,1,19,17) | 4770,61 | 7740 | 109,3939 |
| 8 | (19,1,9,47,26,15,20,14) | 4704,48 | 7740 | 108,1765 |
| 9 | (9,20,33,47,1,19,14,26) | 4728,15 | 6536 | 91,44209 |
| 10 | (19,1,9,47,26,15,20,14) | 4704,48 | 6536 | 144,8164 |
| Best Run | | |  |  |
| 1,3,6,8,10 | (19,1,9,47,26,15,20,14) | 4704,48 | Avg. No. Of iterations=  6674 | Total CPU Time =  1836,499 |

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| Instance: SA:eil76-4 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (40,11,5,1) | 23072,88 | 5184 | 109,4992 |
| 2 | (40,11,5,1) | 23072,88 | 5328 | 110,1797 |
| 3 | (40,11,5,1) | 23072,88 | 7200 | 150,5955 |
| 4 | (40,11,5,1) | 23072,88 | 5904 | 126,4907 |
| 5 | (40,11,5,1) | 23072,88 | 6768 | 149,4134 |
| 6 | (40,11,5,1) | 23072,88 | 4464 | 98,40501 |
| 7 | (40,11,5,1) | 23072,88 | 7740 | 109,3939 |
| 8 | (40,11,5,1) | 23072,88 | 6192 | 145,2881 |
| 9 | (40,11,5,1) | 23072,88 | 5328 | 122,2896 |
| 10 | (40,11,5,1) | 23072,88 | 6048 | 125,2379 |
| Best Run | | |  |  |
| 1,2,3,4,5,6,7,8,9,10 | (40,11,5,1) | 23072,88 | Avg. No. Of iterations=  6016 | Total CPU Time =  1246,79301 |

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| Instance: SA:eil76-6 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (32,47,13,43,38,76) | 17705,87 | 7980 | 165,8411 |
| 2 | (32,47,13,43,38,76) | 17705,87 | 7140 | 148,8856 |
| 3 | (32,47,13,43,38,76) | 17705,87 | 6720 | 144,7156 |
| 4 | (32,47,13,43,38,76) | 17705,87 | 6300 | 135,8713 |
| 5 | (32,47,13,43,38,76) | 17705,87 | 7140 | 156,9318 |
| 6 | (32,47,13,43,38,76) | 17705,87 | 6930 | 156,305 |
| 7 | (38,36,44,1,76,13) | 17941,08 | 6930 | 164,3218 |
| 8 | (38,43,13,32,68,36) | 17969,18 | 8190 | 197,2748 |
| 9 | (32,47,13,43,38,76) | 17705,87 | 6720 | 139,1017 |
| 10 | (32,13,76,65,47,43) | 17953,55 | 8610 | 178,8719 |
| Best Run | | |  |  |
| 1,2,3,4,5,6,9 | (32,47,13,43,38,76) | 17705,87 | Avg. No. Of iterations=  7266 | Total CPU Time =  1588,121 |

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| Instance: SA:eil76-8 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (38,32,13,42,36,62,23,76) | 15293,52 | 7888 | 165,282 |
| 2 | (11,13,44,36,41,72,75,73) | 15290,24 | 10064 | 211,7489 |
| 3 | (13,44,36,76,41,72,11,73) | 15183,5 | 9792 | 211,8132 |
| 4 | (32,38,37,41,73,67,21,13) | 15321,81 | 8704 | 190,8934 |
| 5 | (37,13,38,32,21,41,73,76) | 15270,06 | 10336 | 230,0607 |
| 6 | (37,11,21,72,43,44,13,76) | 15187,63 | 10064 | 241,5189 |
| 7 | (32,41,21,38,37,13,76,1) | 15309,15 | 10880 | 259,3493 |
| 8 | (13,44,36,76,41,72,11,73) | 15183,5 | 8976 | 216,4718 |
| 9 | (32,38,13,76,36,62,23,42) | 15293,52 | 8704 | 236,5682 |
| 10 | (43,11,37,76,13,44,28,72) | 15280,71 | 9520 | 198,1711 |
| Best Run | | |  |  |
| 3,8 | (13,44,36,76,41,72,11,73) | 15183,5 | Avg. No. Of iterations=  9492,8 | Total CPU Time =  2161,8775 |

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| Instance: SA:eil101-4 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (33,93,72,7) | 54147,08 | 8536 | 234,8382 |
| 2 | (33,7,59,56) | 54168,26 | 8342 | 231,0333 |
| 3 | (33,93,72,7) | 54147,08 | 8730 | 243,3372 |
| 4 | (33,93,72,7) | 54147,08 | 8148 | 238,0784 |
| 5 | (33,93,72,7) | 54147,08 | 8342 | 237,0701 |
| 6 | (33,7,59,56) | 54168,26 | 8536 | 242,9275 |
| 7 | (33,93,72,7) | 54147,08 | 8536 | 235,1407 |
| 8 | (33,93,72,7) | 54147,08 | 8536 | 237,2729 |
| 9 | (33,93,72,7) | 54147,08 | 9118 | 250,7681 |
| 10 | (33,93,72,7) | 54147,08 | 8148 | 224,6059 |
| Best Run | | |  |  |
| 1,3,4,5,7,8,9,10 | (33,93,72,7) | 54147,08 | Avg. No. Of iterations=  8497,2 | Total CPU Time =  2375,0723 |

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| Instance: SA:eil101-6 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (20,3,27,98,82,56) | 43341,09 | 9975 | 275,4922 |
| 2 | (27,20,56,82,77,37) | 43585,97 | 13680 | 378,1321 |
| 3 | (20,3,27,98,82,56) | 43341,09 | 11115 | 313,6461 |
| 4 | (20,3,27,98,82,56) | 43341,09 | 10545 | 302,2741 |
| 5 | (20,3,27,98,82,56) | 43341,09 | 10830 | 312,8652 |
| 6 | (9,82,69,93,80,74) | 43869,65 | 11115 | 311,1584 |
| 7 | (82,20,3,27,93,75) | 43476,8 | 11400 | 321,7973 |
| 8 | (82,20,3,27,93,75) | 43476,8 | 11970 | 333,7186 |
| 9 | (20,3,27,98,82,56) | 43341,09 | 14250 | 396,4602 |
| 10 | (20,3,27,98,82,56) | 43341,09 | 9405 | 259,6742 |
| Best Run | | |  |  |
| 1,3,4,5,9,10 | (20,3,27,98,82,56) | 43341,09 | Avg. No. Of iterations=  11428,5 | Total CPU Time =  3205,2184 |

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| --- | --- | --- | --- | --- |
| Instance: SA:eil101-8 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations(L\*epoch) | CPU Time (s) |
| 1 | (82,10,77,9,55,22,93,27) | 36987,91 | 13392 | 371,1924 |
| 2 | (82,10,77,9,55,22,93,27) | 36987,91 | 14136 | 395,873 |
| 3 | (82,10,77,9,55,22,93,27) | 36987,91 | 13764 | 388,9279 |
| 4 | (10,82,77,74,55,27,93,71) | 37060,69 | 11904 | 441,6521 |
| 5 | (10,82,22,55,93,71,27,3) | 36972,23 | 14508 | 156,9318 |
| 6 | (10,98,9,82,22,77,27,55) | 37076,89 | 13020 | 362,2222 |
| 7 | (101,71,22,82,93,10,77,55) | 36993,76 | 14508 | 405,2436 |
| 8 | (10,82,22,55,93,71,27,3) | 36972,23 | 14508 | 403,4411 |
| 9 | (10,82,22,55,93,71,27,3) | 36972,23 | 11160 | 309,621 |
| 10 | (82,10,77,9,55,22,93,27) | 36987,91 | 14508 | 402,3012 |
| Best Run | | |  |  |
| 5,8,9 | (10,82,22,55,93,71,27,3) | 36972,23 | Avg. No. Of iterations=  13540,8 | Total CPU Time =  3637,4063 |

***4.2.2 VNS Results***

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| --- | --- | --- | --- | --- |
| Instance: VNS:eil51-4 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (49,8,4,29) | 7555,76 | 137 | 181,6332 |
| 2 | (49,8,4,29) | 7555,76 | 138 | 183,7242 |
| 3 | (49,8,4,29) | 7555,76 | 138 | 181,7071 |
| 4 | (49,8,4,29) | 7555,76 | 136 | 180,6968 |
| 5 | (49,8,4,29) | 7555,76 | 135 | 181,1479 |
| 6 | (49,8,4,29) | 7555,76 | 138 | 181,934 |
| 7 | (14,17,8,50) | 7555,76 | 138 | 181,1128 |
| 8 | (49,8,4,29) | 7555,76 | 137 | 181,235 |
| 9 | (49,8,4,29) | 7555,76 | 135 | 182,2195 |
| 10 | (8,50,42,47) | 7555,76 | 137 | 181,0932 |
| Best Run | | |  |  |
| 1,2,3,4,5,6,8,9 | (49,8,4,29) | 7555,76 | Avg. No. Of iterations=  136,9 | Total CPU Time =  1816,5037 |

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| Instance: VNS:eil51-6 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (49,26,47,1,29,19) | 6050,81 | 96 | 184,3415 |
| 2 | (20,8,14,19,37,9) | 5836,88 | 97 | 182,271 |
| 3 | (29,49,8,14,19,17) | 5890,23 | 98 | 183,8439 |
| 4 | (19,8,14,20,9,37) | 5836,88 | 96 | 180,2183 |
| 5 | (20,8,37,14,9,19) | 5836,88 | 96 | 180,0717 |
| 6 | (19,37,22,14,9,26) | 6108,35 | 36 | 416,6346 |
| 7 | (8,14,19,35,37,9) | 5930,69 | 23 | 290,7637 |
| 8 | (19,8,14,20,9,37) | 5836,88 | 25 | 293,8351 |
| 9 | (19,8,14,20,9,37) | 5836,88 | 70 | 287,0226 |
| 10 | (19,8,14,20,9,37) | 5836,88 | 26 | 295,7207 |
| Best Run | | |  |  |
| 2,4,5,8,9,10 | (20,8,14,19,37,9) | 5836,88 | Avg. No. Of iterations=  66,3 | Total CPU Time =  2494,7231 |

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| Instance: VNS:eil51-8 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (50,23,47,26,35,1,19,33) | 4934,43 | 19 | 292,2633 |
| 2 | (30,19,12,26,14,1,29,15) | 4911,75 | 19 | 292,2867 |
| 3 | (42,14,50,1,35,47,26,33) | 4788,92 | 19 | 292,254 |
| 4 | (17,19,14,1,9,33,26,20) | 4732,77 | 76 | 183,3238 |
| 5 | (15,14,19,26,47,1,9,20) | 4704,48 | 78 | 186,4048 |
| 6 | (19,10,1,35,50,14,17,26) | 4770,73 | 78 | 187,1797 |
| 7 | (19,10,1,35,50,14,17,26) | 4770,73 | 76 | 182,3675 |
| 8 | (19,10,1,35,50,14,17,26) | 4770,73 | 76 | 183,3165 |
| 9 | (19,20,9,26,1,47,14,15) | 4704,48 | 78 | 186,9266 |
| 10 | (19,20,9,26,1,47,14,15) | 4704,48 | 77 | 184,7244 |
| Best Run | | |  |  |
| 5,9,10 | (19,20,9,26,1,47,14,15) | 4704,48 | Avg. No. Of iterations=  59,6 | Total CPU Time =  2171,0473 |

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| --- | --- | --- | --- | --- |
| Instance: VNS:eil76-4 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (40,5,1,11) | 23072,88 | 101 | 306,3051 |
| 2 | (40,5,1,11) | 23072,88 | 100 | 300,2003 |
| 3 | (32,5,1,53) | 23096,08 | 103 | 308,649 |
| 4 | (40,5,1,11) | 23072,88 | 100 | 305,7628 |
| 5 | (40,5,1,11) | 23072,88 | 99 | 301,2239 |
| 6 | (40,5,1,11) | 23072,88 | 101 | 309,123 |
| 7 | (1,53,5,32) | 23096,08 | 94 | 308,2646 |
| 8 | (1,53,5,32) | 23096,08 | 94 | 304,9079 |
| 9 | (1,53,5,32) | 23096,08 | 83 | 309,4782 |
| 10 | (40,5,1,11) | 23072,88 | 75 | 304,0915 |
| Best Run | | |  |  |
| 1,2,4,5,6,10 | (40,5,1,11) | 23072,88 | Avg. No. Of iterations=  95 | Total CPU Time =  3058,0063 |

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| --- | --- | --- | --- | --- |
| Instance: VNS:eil76-6 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (32,13,76,43,47,38) | 17705,87 | 53 | 303,4146 |
| 2 | (32,37,41,73,46,38) | 18129,57 | 57 | 314,2842 |
| 3 | (32,13,76,43,47,38) | 17705,87 | 66 | 304,9663 |
| 4 | (32,13,76,43,47,38) | 17705,87 | 69 | 313,0765 |
| 5 | (11,34,1,39,44,37) | 18395,48 | 70 | 313,2372 |
| 6 | (32,38,13,76,36,43) | 17719,63 | 68 | 310,4257 |
| 7 | (32,13,76,43,47,38) | 17705,87 | 68 | 305,5853 |
| 8 | (32,38,13,76,36,43) | 17719,63 | 71 | 309,8892 |
| 9 | (38,43,75,32,47,13) | 17825,31 | 71 | 311,3238 |
| 10 | (32,13,76,43,47,38) | 17705,87 | 68 | 308,2014 |
| Best Run | | |  |  |
| 1,3,4,7,10 | (32,13,76,43,47,38) | 17705,87 | Avg. No. Of iterations=  66,1 | Total CPU Time =  3094,4042 |

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| Instance: VNS:eil76-8 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (44,21,37,76,43,39,13,11) | 15204,42 | 52 | 311,4761 |
| 2 | (37,11,43,13,21,32,72,76) | 15268,59 | 46 | 312,1385 |
| 3 | (41,72,11,73,44,36,76,13) | 15183,5 | 46 | 305,6267 |
| 4 | (32,21,76,13,43,37,19,38) | 15354,7 | 46 | 312,4705 |
| 5 | (43,13,38,37,67,44,39,21) | 15445,39 | 46 | 307,852 |
| 6 | (32,21,76,13,43,37,19,38) | 15354,7 | 45 | 300,7682 |
| 7 | (41,72,11,73,44,36,76,13) | 15183,5 | 49 | 300,4076 |
| 8 | (32,37,21,41,73,76,38,13) | 15270,47 | 51 | 311,7343 |
| 9 | (37,44,11,41,21,73,72,46) | 15517,79 | 50 | 317,7916 |
| 10 | (43,21,37,76,11,13,72,44) | 15188,09 | 48 | 301,5273 |
| Best Run | | |  |  |
| 3,7 | (41,72,11,73,44,36,76,13) | 15183,5 | Avg. No. Of iterations=  47,9 | Total CPU Time =  3081,7928 |

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| Instance: VNS:eil101-4 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (56,7,33,59) | 54147,08 | 57 | 310,0654 |
| 2 | (7,33,93,72) | 54147,08 | 58 | 313,9451 |
| 3 | (56,7,59,33) | 54147,08 | 57 | 306,5063 |
| 4 | (7,33,93,72) | 54147,08 | 58 | 308,5545 |
| 5 | (7,33,93,72) | 54147,08 | 59 | 313,3829 |
| 6 | (7,33,93,72) | 54147,08 | 58 | 310,9837 |
| 7 | (7,33,93,72) | 54147,08 | 58 | 314,958 |
| 8 | (7,33,93,72) | 54147,08 | 58 | 314,0591 |
| 9 | (7,33,93,72) | 54147,08 | 58 | 313,7429 |
| 10 | (7,33,93,72) | 54147,08 | 56 | 303,2412 |
| Best Run | | |  |  |
| 2,4,5,6,7,8,9,10 | (7,33,93,72) | 54147,08 | Avg. No. Of iterations=  57,7 | Total CPU Time =  3109,4391 |

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| Instance: VNS:eil101-6 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (98,9,56,10,82,76) | 44020,63 | 38 | 300,4921 |
| 2 | (27,82,3,20,93,75) | 43477,6 | 38 | 304,9517 |
| 3 | (93,76,20,55,82,22) | 44535,07 | 40 | 315,4008 |
| 4 | (9,22,80,93,82,69) | 43969,13 | 40 | 320,8669 |
| 5 | (3,98,20,56,27,82) | 43341,09 | 38 | 302,372 |
| 6 | (82,80,93,69,74,9) | 43871,28 | 41 | 321,3794 |
| 7 | (98,56,83,9,76,62) | 44200,98 | 39 | 305,8036 |
| 8 | (3,98,20,56,27,82) | 43341,09 | 37 | 308,3921 |
| 9 | (82,93,27,72,20,3) | 43699,45 | 11 | 2048,127 |
| 10 | (7,56,3,98,20,27) | 44034,84 | 11 | 1122,213 |
| Best Run | | |  |  |
| 5,8 | (3,98,20,56,27,82) | 43341,09 | Avg. No. Of iterations=  33,3 | Total CPU Time =  5649,9986 |

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| Instance: VNS:eil101-8 | | | | |
| Run No. | Solution (opened facilities) | Obj.  Value | No. of iterations | CPU Time (s) |
| 1 | (4,83,98,20,57,48,27,3) | 38843,65 | 29 | 300,8001 |
| 2 | (101,91,82,10,41,71,55,77) | 38393,08 | 30 | 313,5607 |
| 3 | (93,77,22,71,55,82,10,27) | 36983,64 | 32 | 332,4592 |
| 4 | (27,83,56,68,92,48,9,10) | 37931,91 | 30 | 316,3038 |
| 5 | (9,56,10,42,48,93,27,68) | 38079,8 | 30 | 316,2052 |
| 6 | (68,11,98,25,27,83,9,22) | 37807,32 | 31 | 325,1401 |
| 7 | (27,3,62,71,83,55,22,98) | 37441,52 | 31 | 329,9675 |
| 8 | (10,77,56,71,93,42,27,82) | 37620,2 | 30 | 319,8691 |
| 9 | (37,27,20,3,83,73,48,55) | 38077,4 | 29 | 311,8198 |
| 10 | (82,92,32,65,74,27,3,55) | 38887,2 | 29 | 333,5662 |
| Best Run | | |  |  |
| 2 | (93,77,22,71,55,82,10,27) | 36983,64 | Avg. No. Of iterations=  30,1 | Total CPU Time =  3199,6917 |